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feet long and one foot in diameter, be laid in a horizontal position on the bottom, to what height will the water rise?

Solution by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Let 2θ be the angle at the centre subtended by the chord made by the surface of the water.

Now since radius=6 inches and length=48 inches, area segment=36 $(\theta-\sin\theta\cos\theta)$; volume$=48\times36(\theta-\sin\theta\cos\theta)$; height of water$=6(1-\cos\theta)$. Volume also equals amount of rise in water. Volume$=\{6(1-\cos\theta)-3\}\pi(36)^2$.

$\therefore\ 48\times36(\theta-\sin\theta\cos\theta)=\{6(1-\cos\theta-3\}\pi(36)^2$, $\therefore\ 4\theta-4\sin\theta\cos\theta=9\pi-18\pi\cos\theta$.

Now by double position, $\theta=63^\circ\ 13'\ 55.4''$.

$\therefore\ 6(1-\cos\theta)=3.29772$.

$\therefore$ water rises $3.29772-3=.29772$ inches.

Also solved by *SETH PRATT*, and *H. C. WHITAKER*.

22. Proposed by J. A. TIMMONS, St. Mary's, Kentucky.

Given, the perimeter of a triangle$=100(2s)$, the radius of the inscribed circle $=9(r)$, and the radius of the circumscribed circle$=(R)$; it is required to find (1) the sides of the triangle, (2) the radius of the circle circumscribing the triangle formed by bisecting the exterior angles of the original triangle, (3) the area of the triangle thus formed: all in terms of R, r, s.

I Solution by H. C. WHITAKER, M. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

From the properties of the triangle,

$$a+b+c=2s$$
$$ab+ac+bc=s^2+r^2+4Rr$$
$$abc=4Rrs,$$

whence from the theory of equations, these quantities are the co-efficients of a cubic equation, the roots of which are the three sides. The numerical values are 32, 28.43225 and 39.56775.

Second. The length of the side (passing through A) of the outer triangle is $\dfrac{b\cos\frac{1}{2}C}{\cos\frac{1}{2}B}+\dfrac{c\cos\frac{1}{2}B}{\cos\frac{1}{2}C}$, its opposite angle being $\dfrac{B}{2}+\dfrac{C}{2}$. Dividing the side by twice the sine of this angle gives $\dfrac{b\cos^2\frac{1}{2}C+c\cos^2\frac{1}{2}B}{\sin B\cos^2\frac{1}{2}C+\sin C\cos^2\frac{1}{2}B}$ which by making $c=\dfrac{b\sin C}{\sin B}$ reduces to $b\div\sin B=2R$;—that is, the radius of the circle circumscribed about the outer triangle is twice the radius of the circle circumscribed about the inner triangle. The numerical value is 40.

Third. The length of the side passing through A of the outer triangle is also equal to $\dfrac{S}{\cos\frac{1}{2}B\cos\frac{1}{2}C}$ with similar expressions for the other sides. Now the area of any triangle equals the product of the three sides divided by four times the radius of the circumscribed circle; that is

$$\text{Area}=\frac{S^2}{8R\cos^2\frac{1}{2}A\cos^2\frac{1}{2}B\cos^2\frac{1}{2}C}=2Rs$$

since $\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C=\frac{s}{4R}$. The numerical answer is 2000.

II. Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let ABC represent the triangle, and A', B', C' the centers of escribed circles. Applying opposite resp. to A, B, C; also I centre of inscribed circle. Denote the sides of the triangle $A\ B\ C$ by a, b, c, and its area by $\triangle$; then $a+b+c=2s$ (1), $sr=\triangle$, $\frac{abc}{4\triangle}=\frac{abc}{4sr}=R$, whence $abc=4Rrs$ (2). Squaring the equation $sr=\sqrt{s(s-a)(s-b)(s-c)}$, and putting for $a+b+c$, and abc their respective values, we obtain $ab+ac+bc=r^2+s^2+4Rr$ (3). Consequently the three roots of the cubic

$x^3-2sx^2+(r^2+s^2+4Rr)x-4Rrs=0$ (4) will furnish the three sides of the triangle. For the numerical values we obtain $32, 34\pm\sqrt{31}$.

Denoting the radii of the escribed circles A', B', C' resp. by S, S', S'', we have $A'B'=\frac{S+S'}{\cos\frac{1}{2}C}=\frac{\frac{\triangle}{s-a}+\frac{\triangle}{s-b}}{\cos\frac{1}{2}C}=\frac{\triangle}{\cos\frac{1}{2}C}\ \frac{c}{(s-a)(s-b)}=\frac{c\sqrt{ab}}{\sqrt{(s-a)(s-b)}}$ $=\frac{c}{\sin\frac{1}{2}C}=4R\cos\frac{1}{2}C$ (5) and by analogy $A'C'=4R\cos\frac{1}{2}B$ (6), $B'C'=4R\cos\frac{1}{2}A$ (7). In order to find the area of $\triangle A'B'C'$, we have $sA'B'C''=\frac{1}{2}A'B'$. $A'C'\sin A'=4R^2\cos\frac{1}{2}B\cos\frac{1}{2}C\sin A'=8R^2\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C=8R^2\ \frac{s}{4R}$ $=2Rs$ (8). Denoting the radius of the circle circumscribing $\triangle\ A'C'C''$ by R' we have $R'=\frac{A'B'\times A'C''\times B'C'}{8Rs}=\frac{64R^3\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C}{8Rs}$

$$=\frac{64R^3}{8Rs}\cdot\frac{s}{4R}=2R. \quad (9).$$

PROBLEMS.

32. Proposed by Professor B. F. SINE, Shenandoah Normal College, Reliance, Virginia.

If a given circle is cut by another circle passing through two fixed points the common chord passes through a fixed point.

33. Proposed by T. JOHN COLE, Columbus, Ohio.

A circular field contains 10 acres. A horse is tied to the fence with a rope sufficiently long to graze over one acre. Find length of the rope (1) when the horse is on the inside (2) when he is on the outside of the fence.

Solutions to these problems should be received on or before December 1st.

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